## Digital Electronics

See Boolean Algebra for a description of the category as well as references.

| NAME | $\underset{\text { SYMBOL }}{\text { GRAPHICAL }}$ | ALGEBRAIC EQN | TRUTH TABLE |
| :---: | :---: | :---: | :---: |
| BUFFER |  | $X=A$ | $\begin{array}{ll} \mathrm{A} & \mathrm{X} \\ \hline 0 & 0 \\ 1 & 1 \end{array}$ |
| NOT |  | $X=\bar{A}$ | $\begin{array}{ll} \mathrm{A} & \mathrm{X} \\ \hline 0 & 1 \\ 1 & 0 \end{array}$ |
| AND |  | $X=A B$ or $A^{*} B$ | A B X <br> 0 0 0 <br> 0 1 0 <br> 1 0 0 <br> 1 1 1 |
| NAND |  | $X=\overline{A B}$ or $\overline{A^{*} B}$ | $\begin{array}{lll} \hline \mathrm{A} & \mathrm{~B} & \mathrm{X} \\ \hline 0 & 0 & 1 \\ 0 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \\ \hline \end{array}$ |
| OR |  | $X=A+B$ | A B X <br> 0 0 0 <br> 0 1 1 <br> 1 0 1 <br> 1 1 1 |
| NOR |  | $X=\overline{A+B}$ | A B X <br> 0 0 1 <br> 0 1 0 <br> 1 0 0 <br> 1 1 0 |
| $\begin{aligned} & \text { EXCLUSIVE-OR } \\ & \text { (XOR) } \end{aligned}$ |  | $X=A \oplus B$ | A B X <br> 0 0 0 <br> 0 1 1 <br> 1 0 1 <br> 1 1 0 |
| EQUIVALENCE (XNOR) |  | $X=\overline{A \oplus B}$ | $\begin{array}{lll} \hline \mathrm{A} & \mathrm{~B} & \mathrm{X} \\ \hline 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \\ 1 & 1 & 1 \\ \hline \end{array}$ |

Sample Problems

Find all ordered 4-tuples $(A, B, C, D)$, which make the following circuit FALSE:


The circuit translates to the following Boolean expression:

$$
(\overline{C+D}+\bar{B}) \oplus(\bar{A} B) \oplus(\overline{C+D)}
$$

The following table has the following headings: H 1 is $\overline{(C+D)}, \mathrm{H} 2$ is $\mathrm{H} 1+\bar{B}, \mathrm{H} 3$ is $\bar{A} B, \mathrm{H} 4$ is $\mathrm{H} 2 \oplus \mathrm{H} 3$ and H 5 is $\mathrm{H} 4 \oplus \mathrm{H} 1$, the final expression.

| $\mathbf{A}$ | $\mathbf{B}$ | $\mathbf{C}$ | $\mathbf{D}$ | $\mathbf{H 1}$ | $\mathbf{H 2}$ | $\mathbf{H 3}$ | $\mathbf{H 4}$ | H5 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 0 | 0 | 0 | 0 | 1 | 1 | 0 | 1 | 0 |
| 0 | 0 | 0 | 1 | 0 | 1 | 0 | 1 | 1 |
| 0 | 0 | 1 | 0 | 0 | 1 | 0 | 1 | 1 |
| 0 | 0 | 1 | 1 | 0 | 1 | 0 | 1 | 1 |
| 0 | 1 | 0 | 0 | 1 | 1 | 1 | 0 | 1 |
| 0 | 1 | 0 | 1 | 0 | 0 | 1 | 1 | 1 |
| 0 | 1 | 1 | 0 | 0 | 0 | 1 | 1 | 1 |
| 0 | 1 | 1 | 1 | 0 | 0 | 1 | 1 | 1 |
| 1 | 0 | 0 | 0 | 1 | 1 | 0 | 1 | 0 |
| 1 | 0 | 0 | 1 | 0 | 1 | 0 | 1 | 1 |
| 1 | 0 | 1 | 0 | 0 | 1 | 0 | 1 | 1 |
| 1 | 0 | 1 | 1 | 0 | 1 | 0 | 1 | 1 |
| 1 | 1 | 0 | 0 | 1 | 1 | 0 | 1 | 0 |
| 1 | 1 | 0 | 1 | 0 | 0 | 0 | 0 | 0 |
| 1 | 1 | 1 | 0 | 0 | 0 | 0 | 0 | 0 |
| 1 | 1 | 1 | 1 | 0 | 0 | 0 | 0 | 0 |

Thus, the 4-tuples $(0,0,0,0),(1,0,0,0),(1,1,0,0),(1,1,0,1),(1,1,1,0)$, and $(1,1,1,1)$ all make the circuit FALSE.

Find all ordered triplets $(A, B, C)$ which make the following circuit FALSE:


The circuit translates to the following Boolean expression: $\overline{A B}+C$. To find when this is
FALSE we can equivalently find when the $\overline{\overline{A B}+C}$ is TRUE. We can simplify this by applying DeMorgan's Law and cancelling the double not over $A B$ to yield $A B \bar{C}$. This is TRUE when all three terms are TRUE, which happens for $(1,1,0)$.

